

TD 22):

Ex 8). F et G deux sous- K -es de E .

$$\text{On a, } \dim(F) + \dim(G) \geq \dim(E).$$

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$$\text{Mq. } F \cap G \neq \{\vec{0}_E\}$$

LEBRETON Ruben

$F+G$ est un s.e.v. de E .

RANDRIANALOW Georges

$$\text{Donc } \dim(F+G) \leq \dim(E).$$

D'après la formule de Grassmann,

$$\dim(F) + \dim(G) - \dim(F \cap G) \leq \dim(E).$$

$$\Leftrightarrow \underline{\dim(F) + \dim(G)} - \dim(E) \leq \dim(F \cap G).$$
$$> \dim(E)$$

$$\text{Donc, } \dim(F \cap G) > 0 \quad \checkmark$$

$$\text{Ainsi } \underline{F \cap G \neq \{\vec{0}_E\}}. \quad \checkmark$$

Ex 15) E K -es de dimension finie et $(f, g) \in \mathcal{L}(E)^2$.

$$\text{Mq. } |\operatorname{rg}(f) - \operatorname{rg}(g)| \leq \operatorname{rg}(f+g) \leq \operatorname{rg}(f) + \operatorname{rg}(g).$$

$$\text{Mq. } \operatorname{rg}(f+g) \leq \operatorname{rg}(f) + \operatorname{rg}(g).$$

$$\text{On a } \operatorname{Im}(f+g) \subset \operatorname{Im}(f) + \operatorname{Im}(g). \quad \checkmark$$

$$\text{Donc, } \dim(\operatorname{Im}(f+g)) \leq \dim(\operatorname{Im}(f)) + \dim(\operatorname{Im}(g)). \quad \checkmark$$

$$\text{Donc } \underline{\operatorname{rg}(f+g) \leq \operatorname{rg}(f) + \operatorname{rg}(g)}. \quad \checkmark$$

$$\text{On a } f = f+g-g.$$

$$\text{Donc, } \operatorname{rg}(f) = \operatorname{rg}(f+g-g) \leq \operatorname{rg}(f+g) + \underbrace{\operatorname{rg}(-g)}_{=\operatorname{rg}(g)} \quad \checkmark$$

$$\text{Ainsi, } \operatorname{rg}(f) \leq \operatorname{rg}(f+g) + \operatorname{rg}(g) \quad \checkmark$$

$$\text{Donc, } \operatorname{rg}(f) - \operatorname{rg}(g) \leq \operatorname{rg}(f+g) \quad \checkmark$$

Par symétrie du raisonnement, on obtient

$$\operatorname{rg}(g) - \operatorname{rg}(f) \leq \operatorname{rg}(f+g) \quad \checkmark$$

$$\text{Ainsi } |\operatorname{rg}(f) - \operatorname{rg}(g)| \leq \operatorname{rg}(f+g). \quad \checkmark$$

$$\text{En conclusion, } |\operatorname{rg}(f) - \operatorname{rg}(g)| \leq \operatorname{rg}(f+g) \leq \operatorname{rg}(f) + \operatorname{rg}(g). \quad \checkmark$$