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Exercice 5 -

$g_1(x) = \ln(\sin(x)) \underset{0}{\sim} \ln(x)$

Il faut le prouver : composer des équivalents est interdit a priori.

et $\sin(x) \underset{0}{\sim} x$

$g_2(x) = -\mathrm{cotan}(x) \cdot \sin^2(nx) \underset{0}{\sim} -\frac{1}{x} \cdot n^2x^2 \underset{0}{\sim} -n^2x$ ✓

Car $\sin^2(nx) \underset{0}{\sim} (nx)^2$ ✓

$\mathrm{cotan}(x) = \frac{1}{\tan(x)}$ ✓

et $\tan(x) \underset{0}{\sim} x$ donc $-\mathrm{cotan}(x) \underset{0}{\sim} -\frac{1}{x}$ ✓

$g_3(x) = \frac{2\arctan(x) - \arctan(x)}{x^{3/2}}$

$2\arctan(x) = 2\left(x - \frac{x^3}{3} + o(x^3)\right)$ Vous avez oublié le 2

$-\arctan(2x) = -\left(2x - \frac{8x^3}{3} + o(x^3)\right) = -2x + \frac{8x^3}{3} + o(x^3)$

$2\arctan(x) - \arctan(2x) = 2\left(x - \frac{x^3}{3}\right) - 2x + \frac{8x^3}{3} + o(x^3) = \cancel{-x} + \frac{\cancel{7}x^3}{3} + o(x^3)$

$g_3(x) = \left(-x + \frac{7x^3}{3} + o(x^3)\right)x^{-3/2} = -x^{-1/2} + \frac{7x^{3/2}}{3} + o(x^{3/2})$

$g_4(\cancel{x}\ t) = \frac{\sin(t)}{t^\alpha} \underset{0}{\sim} \frac{t}{t^\alpha} \underset{0}{\sim} t^{1-\alpha}$ ✓

$g_5(x) = \frac{\ln(x)}{(1+x)^3}$

en $+\infty$ $(1+x)^3 \underset{+\infty}{\sim} x^3$ ✓ $g_5(x) \underset{+\infty}{\sim} \frac{\ln(x)}{x^3}$ ✓

en 0 $(1+x)^3 \sim 1$ $g_5(x) \sim \ln(x)$ ✓

$g_6(x) = \frac{\sin(x)}{2+\cos(x)} - \frac{x}{3} = \frac{\sin(x)}{3-1+\cos(x)} - \frac{x}{3}$

$\sin(x) \underset{0}{\sim} x$

$2+\cos(x) \underset{0}{\sim} 3 - \frac{x^2}{2}$ et $3 - \frac{x^2}{2} = 3\left(1 - \frac{x^2}{6}\right)$ on pose $u = \frac{-x^2}{6} \underset{0}{\to} 0$

donc $\frac{1}{2+\cos(x)} = \frac{1}{3(1+u)} = \frac{1}{3}\left(1 - u + u^2 - u^3 + o(u^3)\right)$

$= \frac{1}{3}\left(1 + \frac{x^2}{6} + o(x^2)\right)$ ✓

$= \frac{1}{3} + \frac{x^2}{18} + o(x^2)$ ✓

donc $g_6(x) = x\left(\frac{1}{3} + \frac{x^2}{18} + o(x^2)\right) - \frac{x}{3} = \frac{x^3}{18} + o(x^3)$ ✓ donc $g_6(x) \underset{x\to 0}{\sim} \frac{x^3}{18}$