

Exercise 12

$$P_3 = X^8 + 1.$$

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$$X^8 + 1 = \prod_{k=1}^8 (X - e^{\frac{(2k+1)\pi}{8}}) \quad \checkmark$$

$$\begin{aligned} &= (X - e^{i\frac{3\pi}{8}})(X - e^{i\frac{5\pi}{8}})(X - e^{i\frac{7\pi}{8}})(X - e^{i\frac{9\pi}{8}}) \\ &\quad (X - e^{i\frac{11\pi}{8}})(X - e^{i\frac{13\pi}{8}})(X - e^{i\frac{15\pi}{8}})(X - e^{i\frac{17\pi}{8}}) \\ &= (X - e^{i\frac{3\pi}{8}})(X - e^{\overline{i\frac{3\pi}{8}}})(X - e^{i\frac{5\pi}{8}})(X - e^{\overline{i\frac{5\pi}{8}}}) \\ &\quad (X - e^{i\frac{7\pi}{8}})(X - e^{\overline{i\frac{7\pi}{8}}})(X - e^{i\frac{9\pi}{8}})(X - e^{\overline{i\frac{9\pi}{8}}}) \end{aligned}$$

$$= (X^2 - 2\cos(\frac{3\pi}{8}) + 1)(X^2 - 2\cos(\frac{5\pi}{8}) + 1)$$

$$(X^2 - 2\cos(\frac{7\pi}{8}) + 1)(X^2 - 2\cos(\frac{9\pi}{8}) + 1) \quad \checkmark$$

$$\cos(X - \omega)(X - \bar{\omega}) = X^2 - \underbrace{(\omega + \bar{\omega})}_{= 2\cos(\theta)} + \underbrace{\omega\bar{\omega}}_{= 1}$$

$$P_5 = X^n - 1 = \prod_{k=0}^{n-1} (X - e^{i\frac{2k\pi}{n}})$$